2.13H To Market with Matrices
A Solidify Understanding Task

Carlos learned about matrices when Elvira, the manager of the school cafeteria, was asked to substitute teach during one of the last days of school before summer vacation. Now that he has worked out a strategy for solving systems of equations by elimination of variables, he is wondering if matrices can help him keep track of his work.

Carlos is reconsidering the following scenario from "Shopping for Cats and Dogs", while trying to record his thinking using matrices.

> *One week Carlos purchased 6 dog leashes and 6 cat brushes for $45.00 for Clarita to use while pampering the pets. Later in the summer he purchased 3 additional dog leashes and 2 cat brushes for $19.00. What is the price of each item?*

Carlos realizes that he can represent this scenario using the following matrix:

$$\begin{array}{c} \\ purchase\ 1 \\ purchase\ 2 \end{array} \begin{array}{ccc} leashes & brushes & total \\ \left[\begin{array}{ccc} 6 & 6 & \$45.00 \\ 3 & 2 & \$19.00 \end{array}\right] \end{array}$$

He also realizes that he can represent the cost of each item with a matrix that looks like this:

$$\begin{array}{c} \\ purchase\ 1 \\ purchase\ 2 \end{array} \begin{array}{ccc} leashes & brushes & total \\ \left[\begin{array}{ccc} 1 & 0 & \$4.00 \\ 0 & 1 & \$3.50 \end{array}\right] \end{array}$$

So, now he is trying to find a sequence of matrices that can fill in the gaps between the first matrix and the last. He knows from his previous work with solving systems of equations that he can do any of the following manipulations with equations—and he realizes that each of these manipulations would give him a new row of numbers in a corresponding matrix.

- Replace an equation in the system with a constant multiple of that equation
- Replace an equation in the system with the sum or difference of the two equations
- Replace an equation with the sum of that equation and a multiple of the other

1. Help Carlos find a sequence of matrices that starts with the matrix that represents the original purchases, and ends with the matrix that represents purchasing one leash or purchasing one brush. For each matrix in your sequence, write out the justification that allows you to write that matrix based on the three manipulations we can perform on the equations in a system. For example, the following matrix transformation can be justified by writing "I replaced the first row of the matrix by multiplying the first row by $\frac{1}{6}$."

$$\begin{bmatrix} 6 & 6 & 45.00 \\ 3 & 2 & 19.00 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 7.50 \\ 3 & 2 & 19.00 \end{bmatrix}$$

2. Find and justify a sequence of matrices that could be used to solve the following scenario.

   One week Carlos tried out cheaper brands of cat and dog food. On Monday he purchased 3 small bags of cat food and 5 small bags of dog food for $22.75. Because he went through the small bags quite quickly, he had to return to the store on Thursday to buy 2 more small bags of cat food and 3 more small bags of dog food, which cost him $14.25. Based on this information, can you figure out the price of each bag of the cheaper cat and dog food?