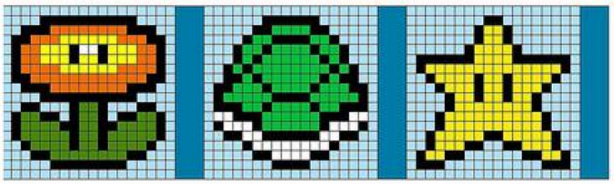
2.3 Building the Perfect Square

A Solidify Understanding Task

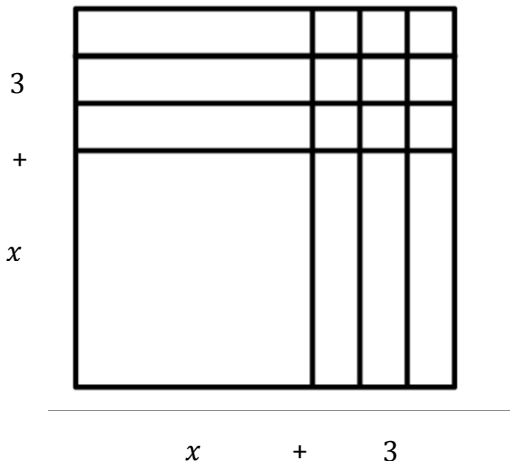


Part 1: Quadratic Quilts

Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that the can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. If Optima adds 3 inches to the side of the square, what is the area of the square?

When Optima draws a pattern for the square in problem #1, it looks like this:



2. Use both the diagram and the equation, $A(x) = (x + 3)^2$ to explain why the area of the quilt block square, $A(x)$, is also equal to the $x^2 + 6x + 9$.

The customer service representatives at Optima's shop work with customer orders and write up the orders based on the area of the fabric needed for the order. As you can see from problem #2 there are two ways that customers can call in and describe the area of the quilt block. One way describes the length of the sides of the block and the other way describes the areas of each of the four sections of the block.

For each of the following quilt blocks, draw the diagram of the block and write two equivalent equations for the area of the block.

1. Block with side length: $x + 2$.

2. Block with side length: $x + 1$.

3. What patterns do you notice when you relate the diagrams to the two expressions for the area?

4. Optima likes to have her little dog, Clementine, around the shop. One day the dog got a little hungry and started to chew up the orders. When Optima found the orders, one of them was so chewed up that there were only partial expressions for the area remaining. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

   a. $x^2 + 4x$

   b. $x^2 + 6x$

   c. $x^2 + 8x$

   d. $x^2 + 12x$

5. If $x^2 + bx + c$ is a perfect square, what is the relationship between b and c? How do you use b to find c, like in problem 6?

   Will this strategy work if b is negative? Why or why not?

   Will the strategy work if b is an odd number? What happens to c if b is odd?

Sometimes a customer orders more than one quilt block of a given size. For instance, when a customer orders 4 blocks of the basic size, the customer service representatives write up an order for $A(x) = 4x^2$.

6. What would they write if the order was for 2 blocks that are 1 inch longer than the basic block? Write the area function in two equivalent forms. Verify your algebra using a diagram.

Part 2: Quilts and Quadratic Graphs

Optima's niece, Jenny works in the shop, taking orders and drawing quilt diagrams. When the shop isn't too busy, Jenny pulls out her math homework and works on it. One day, she is working on graphing parabolas and notices that the equations she is working with look a lot like an order for a quilt block. For instance, Jenny is supposed to graph the equation: $y = (x - 3)^2 + 4$. She thinks, "That's funny. This would be an order where the length of the standard square is reduced by 3 and then we add a little piece of fabric that has as area of 4. We don't usually get orders like that, but it still makes sense. I better get back to thinking about parabolas. Hmmm…"

7. Fully describe the parabola that Jenny has been assigned to graph.

8. Jenny returns to her homework, which is about graphing quadratic functions. Much to her dismay, she finds that she has been given: $y = x^2 - 6x + 9$. "Oh dear", thinks Jenny. "I can't tell where the vertex is or any of the transformations of the parabola in this form. Now what am I supposed to do?"

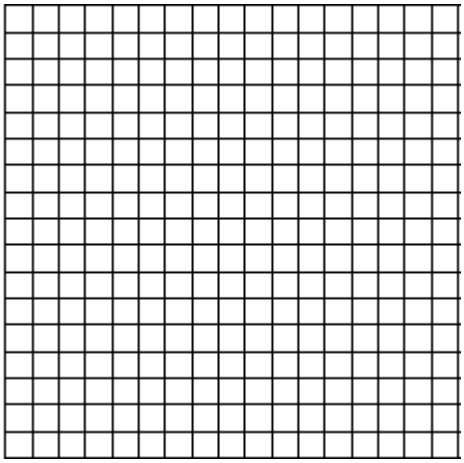
   "Wait a minute—is this the area of a perfect square?" Use your work from Part 1of this task to answer Jenny's question and justify your answer.

9. Jenny says, "I think I've figured out how to change the form of my quadratic equation so that I can graph the parabola. I'll check to see if I can make my equation a perfect square." Jenny's equation is: $y = x^2 - 6x + 9$. Change the form of the equation, find the vertex, and graph the parabola.

   a. $y = x^2 - 6x + 9$ New form of the equation: _____

   b. Vertex of the parabola: _____

   c. Graph (with at least 3 accurate points on each side of the line of symmetry):

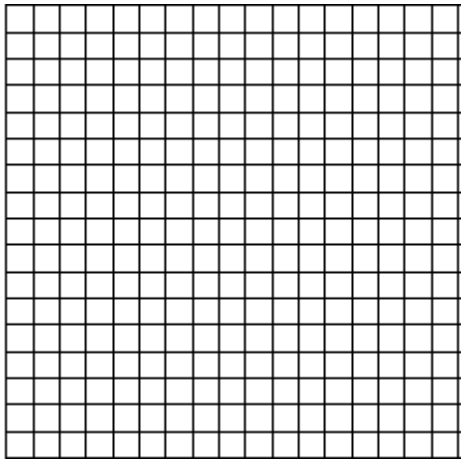


10. The next quadratic to graph on Jenny's homework is $y = x^2 + 4x + 2$. Does this expression fit the pattern for a perfect square? Why or why not?

a. Use an area model to figure out how to complete the square so that the equation can be written in vertex form, $y = a(x - h)^2 + k$.

b. Is the equation you have written equivalent to the original equation? If not, what adjustments need to be made? Why?

c. Identify the vertex and graph the parabola with three accurate points on both sides of the line of symmetry.



11. Jenny hoped that she wasn't going to need to figure out how to complete the square on an equation where b is an odd number. Of course, that was the next problem. Help Jenny to find the vertex of the parabola for this quadratic function:

$g(x) = x^2 + 7x + 10$

12. Jenny's last quadratic function to graph is $f(x) = 2x^2 + 12x + 13$. She draws the following diagram and says, "I'm not sure how this helps me. I don't see how to make this a square." Help Jenny to complete the square and find the vertex of the parabola using either the diagram or the equation.