3.11H Quadratic Quandaries

A Develop and Solidify Understanding Task

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In the task *Curbside Rivalry* Carlos and Clarita were trying to decide how much they should charge for a driveway mascot. Here are the important details of what they had to consider.

- *Surveys show the twins can sell 100 driveway mascots at a cost of $20, and they will sell 10 fewer mascots for each additional $5 they charge.*

- *The twins estimate that the cost of supplies will be $250 and they would like to make $2000 in profit from selling driveway mascots. Therefore, they will need to collect $2250 in revenue.*

This information led Carlos and Clarita to write and solve the quadratic equation $(100-10x)(20+5x)=2250$.

1. Either review your work from *Curbside Rivalry* or solve this quadratic equation for *x* again.

2. What do your solutions for *x* mean in terms of the story context?

3. How would your solution change if this had been the question Carlos and Clarita had asked: "How much should we charge if we want to collect at least $2250 in revenue?"

4. What about this question: "How much should we charge if we want to maximize our revenue?"

As you probably observed, the situation represented in question 3 didn't have a single solution, since there are many different prices the twins can charge to collect more than $2250 in revenue. Sometimes our questions lead to quadratic inequalities rather than quadratic equations.

Here is another quadratic inequality based on your work on *Curbside Rivalry.*

5. Carlos and Clarita want to design a logo that requires less than 48 in^2 of paint, and fits inside a rectangle that is 8 inches longer than it is wide. What are the possible dimensions of the rectangular logo?

Again question 5 has multiple answers, and those answers are restricted by the context. Let's examine the inequality you wrote for question 5, but not restricted by the context.

6. What are the solutions to the inequality $x(x+8) < 48$?

7. How might you support your answer to question 6 with a graph or a table?

Here are some more quadratic inequalities without contexts. Show how you might use a graph, along with algebra, to solve each of them.

8. $x^2 + 3x - 10 \geq 0$

9. $2x^2 - 5x < 12$

10. $x^2 - 4 \leq 4x + 1$

Carlos and Clarita both used algebra and a graph to solve question 10, but they both did so in different ways. Illustrate each of their methods with a graph and with algebra.

11. Carlos: "I rewrote the inequality to get 0 on one side and a factored form on the other. I found the zeroes for each of my factors. To decide what values of *x* made sense in the inequality I also sketched a graph of the quadratic function that was related to the quadratic expression in my inequality. I shaded solutions for *x* based on the information from my graph."

12. Clarita: "I graphed a linear function and a quadratic function related to the linear and quadratic expressions in the inequality. From the graph I could estimate the points of intersection, but to be more exact I solved the quadratic equation $x^2 - 4 = 4x + 1$ by written an equivalent equation that had 0 on one side. Once I knew the *x*-values for the points of intersection in the graph, I could shade solutions for *x* that made the inequality true."

Carlos and Clarita have decided to create 3-D mascots out of clay for their customers who want them. They want the mascot to fit within a rectangular box with a volume that is no more than 96 in^2 and whose width is 2 inches shorter than its length, and whose height is 8 inches more than its length.

Carlos writes this inequality to represent the box's description: $x(x-2)(x+8) \leq 96$

With the help of his cousin who is in advanced mathematics he is able to rewrite this inequality in an equivalent factored form that has 0 on one side of the inequality: $(x-4)(x+4)(x+8) \leq 0$

Because Carlos doesn't know how to graph cubic polynomials any better than he can factor them, he is wondering how his work with quadratic inequalities might help his solve this cubic inequality.

13. Devise a strategy based on your work with quadratic inequalities that could be used to solve this cubic inequality with three factors: $(x-4)(x+4)(x+8) \leq 0$

14. Use the solutions to this cubic inequality to determine the dimensions of rectangular boxes that meet their criteria.

15. Here is the algebra work produced by Carlos' cousin. Explain each step in the process that led from Carlos' inequality to his cousin's.

$$x(x-2)(x+8) \leq 96$$
$$x(x^2+6x-16) \leq 96$$
$$x^3+6x^2-16x \leq 96$$
$$x^3+6x^2-16x-96 \leq 0$$
$$x^2(x+6)-16(x+6) \leq 0$$
$$(x^2-16)(x+6) \leq 0$$
$$(x-4)(x+4)(x+6) \leq 0$$

3.11H Quadratic Quandaries – Teacher Notes

A Develop and Solidify Understanding Task

Purpose: The purpose of this task is to develop a strategy for solving quadratic inequalities and extend this strategy to higher-degree polynomials when the factors are known. The context of the task gives students an opportunity to engage in mathematical modeling: students will use mathematical models, in this case quadratic and cubic inequalities, to model various contextualized situations. The solutions to the inequalities then have to be interpreted in terms of what they mean in the situations. That is, the solutions for *x* in the inequalities are not the answers to the questions being asked in the situations—rather they provide information from which those questions can be answered. Students will have to keep track of the meaning of the variables as they work through these problems.

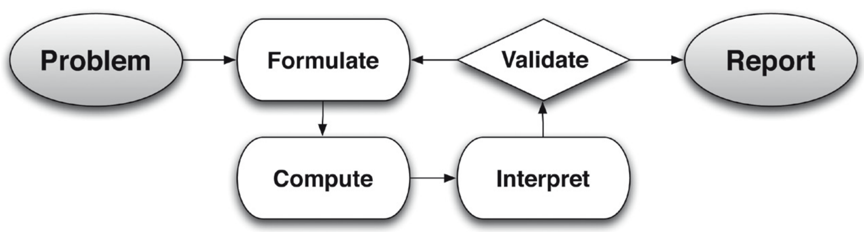
Core Standards Focus:

Utah Secondary II Honors Standard: Solve polynomial and rational inequalities in one variable.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Related Standards: High School Modeling Standard



Launch (Whole Class):
Use question 1 to remind students of the *Curbside Rivalry* context and the work they did to solve the quadratic equation $(100-10x)(20+5x)=2250$ by first multiplying out the binomials, subtracting to get 0 on one side of the equation and then factoring the resulting quadratic into new factors whose product equals 0. Remind them that this meant the solutions to the equation would result when either factor equaled 0. One this strategy for solving quadratics is firmly in place, set students to work on the task questions, which will involve solving quadratic and cubic inequalities.

Explore (Small Group):
For question 2, make sure students understand that *x* represents the number of $5 increments in the price, and therefore, once we have solved for *x* we have to substitute the solutions back into the

expression for price to determine the amount we should charge if we want to collect $2250 in revenue. On question 3 students should note that there is an interval of prices for which the revenue is greater than $2250. Again, x will not give us the prices, but will help us to find the prices we should charge. Make sure students are making sense of how to use the algebraic model to answer questions from the context. Question 4 cannot be answered by solving an equation or inequality. Rather, it requires changing the form of the quadratic to vertex form so the maximum value can be identified.

The table and graph requested in question 6 should lend support to how to interpret the algebraic inequality written in question 5. Listen for students who are beginning to articulate a strategy for solving quadratic inequalities:

> Get 0 on one side of the inequality and factor the quadratic expression on the other side. Find the zeroes of the factors. Determine the sign of the values of the quadratic expression between the zeroes by either testing specific points or referring to the graph. Select the intervals that yield the appropriate signs for the values—positive values for expressions that are greater than 0 or negative values for expressions that are less than 0.

Questions 5 and 6 are a good place to start the modeling conversation. Make sure the students understand what the factors represent in the inequality $x(x+8) \leq 48$ relative to the story context. While this inequality represents the story perfectly, it is not easy to solve the inequality while written in this form. The preferred form for solving the inequality algebraically, $x^2 + 8x - 48 \leq 0$, obscures the context, but allows us to work algebraically towards a solution. The solution obtained algebraically, $-12 \leq x \leq 4$, solves the inequality, but does not answer the question asked by the scenario, since in the scenario x represents a length, and therefore, cannot be negative. So the solution to question asked by the story context is $0 \leq x \leq 4$. Make sure that students understand that the work of this task requires both the algebraic work of solving polynomial inequalities, as well as the work of interpreting those solutions within the context for which the inequality was written.

Questions 8-10 give students opportunities to solidify their strategy for solving polynomial inequalities. Watch for alternative strategies in addition to the one described above. You might want to discuss the strategies that have emerged during student work on questions 2-10 before assigning the remaining portion of the task.

Questions 11 and 12 illustrate two alternative strategies for solving quadratic inequalities. Questions 13 and 14 ask students to adapt one of these strategies (Carlos' strategy in question 11) to a situation involving a cubic polynomial. Make sure students focus on both the strategy and the modeling context. As probing questions such as, "How does your solution to the cubic inequality help you determine the dimensions of boxes that fit the required conditions?"

Discuss (Whole Class):
The whole class discussion should focus on student strategies for solving the quadratic inequalities in questions 8-10, and which strategy they used for the cubic inequality in question 13. Once all issues with the methods for solving polynomial inequalities has been resolved, turn the discussion to the modeling issues. Make sure that students have interpreted their solutions to the inequalities to get appropriate answers for the story contexts in questions 5 and 14.

Question 15 introduces some interesting algebra: factoring a cubic polynomial by grouping. While this algebra should be accessible to your students, it is not an expected procedure for this course.

Aligned Ready, Set, Go: Solving Quadratic and Other Equations 3.11H