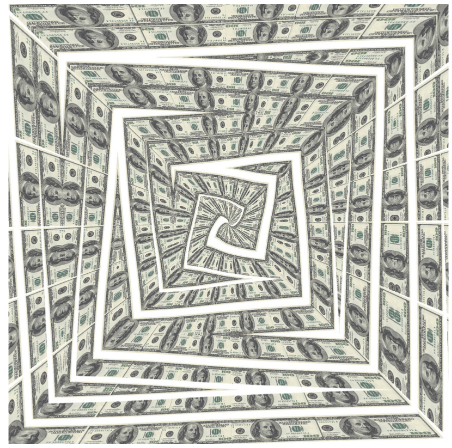
3.3 More Interesting!

A Solidify Understanding Task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Carlos now knows he can calculate the amount of interest earned on an account in smaller increments than one full year. He would like to determine how much money is in an account each month that earns 5% annually with an initial deposit of $300.

He starts by considering the amount in the account each month during the first year. He knows that by the end of the year the account balance should be $315, since it increases 5% during the year.

1. Complete the table showing what amount is in the account each month during the first twelve months.

deposit											1 year
$300											$315

2. What number did you multiply the account by each month to get the next month's balance?

Carlos knows the exponential equation that gives the account balance for this account on an annual basis is $A = 300(1.05)^t$

Based on his work finding the account balance each month, Carlos writes the following equation for the same account: $A = 300(1.05^{1/12})^{12t}$.

3. Verify that both equations give the same results. Using the properties of exponents, explain why these two equations are equivalent.

4. What is the meaning of the $12t$ in this equation?

Carlos shows his equation to Clarita. She suggests his equation could also be approximated by $A = 300(1.004)^{12t}$, since $(1.05)^{1/12} \approx 1.004$. Carlos replies, "I know the 1.05 in the equation $A = 300(1.05)^{t}$ means I am earning 5% interest annually, but what does the 1.004 mean in this equation?"

5. Answer Carlos' question. What does the 1.004 mean in $A = 300(1.004)^{12t}$?

The properties of exponents can be used to explain why $[(1.05)^{1/12}]^{12t} = 1.05^{t}$. Here are some more examples of using the properties of exponents with rational exponents. For each of the following, simplify the expression using the properties of exponents, and explain what the expression means in terms of the context.

6. $(1.05)^{1/12} \cdot (1.05)^{1/12} \cdot (1.05)^{1/12}$

7. $[(1.05)^{1/12}]^{6}$

8. $(1.05)^{-1/12}$

9. $(1.05)^{2} \cdot (1.05)^{1/4}$

10. $\dfrac{(1.05)^{2}}{(1.05)^{1/2}}$

Carlos and Clarita now have two equations representing the balance of their 5% account after *t* years: $A = 300(1.05)^t$ and $A = 300(1.05^{1/12})^{12t}$. In both of these equations *t* represents the amount of time the money has been in the account in terms of *years*.

Carlos and Clarita know they can use their equations for fractions of a year by expressing *t* in terms of a portion of a year, for example, using $t = 2.5$ for two and one-half years or $t = 1/12$ for one month. They are wondering if they can write an equation that would find the account balance in terms of *t* months or *t* days.

11. Write an equation that will find the account balance in terms of *t* months.

12. Write an equation that will find the account balance in terms of *t* days.

13. The account balance is currently $382.88. Write an equation that will find the account balance *t* months ago.