3.6 Curbside Rivalry

A Solidify Understanding Task

©2013 www.flickr.com/photos/NAPARAZZI

Carlos and Clarita have a brilliant idea for how they will earn money this summer. Since the community in which they live includes many high schools, a couple of universities, and even some professional sports teams, it seems that everyone has a favorite team they like to root for. In Carlos' and Clarita's neighborhood these rivalries take on special meaning, since many of the neighbors support different teams. They have observed that their neighbors often display handmade posters and other items to make their support of their favorite team known. The twins believe they can get people in the neighborhood to buy into their new project: painting team logos on curbs or driveways.

For a small fee, Carlos and Clarita will paint the logo of a team on a neighbor's curb, next to their house number. For a larger fee, the twins will paint a mascot on the driveway. Carlos and Clarita have designed stencils to make the painting easier and they have priced the cost of supplies. They have also surveyed neighbors to get a sense of how many people in the community might be interested in purchasing their service. Here is what they have decided, based on their research.

- *A curbside logo will require 48 in^2 of paint*
- *A driveway mascot will require 16 ft^2 of paint*
- *Surveys show the twins can sell 100 driveway mascots at a cost of $20, and they will sell 10 fewer mascots for each additional $5 they charge*

1. If a curbside logo is designed in the shape of a square, what will its dimensions be?

A square logo will not fit nicely on a curb, so Carlos and Clarita are experimenting with different types of rectangles. They are using a software application that allows them to stretch or shrink their logo designs to fit different rectangular dimensions.

2. Carlos likes the look of the logo when the rectangle in which it fits is 8 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write a quadratic equation that represents these requirements.

3. Clarita prefers the look of the logo when the rectangle in which it fits is 13 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write a quadratic equation that represents these requirements.

Your quadratic equations on the previous two problems probably started out looking like this: $x(x+n)=48$ where *n* represents the number of inches the rectangle is longer than it is wide. The expression on the left of the equation could be multiplied out to get and equation of the form $x^2+nx=48$. If we subtract 48 from both sides of this equation we get $x^2+nx-48=0$. In this form, the expression on the left looks more like the quadratic functions you have been working with in previous tasks, $y=x^2+nx-48$.

4. Consider Carlos' quadratic equation where *n* = 8, so $x^2+8x-48=0$. How can we use our work with quadratic functions like $y=x^2+8x-48$ to help us solve the quadratic equation $x^2+8x-48=0$? Describe at least two different strategies you might use, and then carry them out. Your strategies should give you solutions to the equation as well as a solution to the question Carlos is trying to answer in #2.

5. Now consider Clarita's quadratic equation where *n* = 13, so $x^2+13x-48=0$. Describe at least two different strategies you might use to solve this equation, and then carry them out. Your strategies should give you solutions to the equation as well as a solution to the question Clarita is trying to answer in #3.

6. After much disagreement, Carlos and Clarita agree to design the curbside logo to fit in a rectangle that is 6 inches longer than it is wide. What would the dimensions of the curbside logo need to be to fit in this type of rectangle? As part of your work, write and solve a quadratic equation that represents these requirements.

7. What are the dimensions of a driveway mascot if it is designed to fit in a rectangle that is 6 feet longer than it is wide? (See the requirements for a driveway mascot given in the bulleted list above.) As part of your work, write and solve a quadratic equation that represents these requirements.

8. What are the dimensions of a driveway mascot if it is designed to fit in a rectangle that is 10 feet longer than it is wide? (See the requirements for a driveway mascot given in the bulleted list above.) As part of your work, write and solve a quadratic equation that represents these requirements.

Carlos and Clarita are also examining the results of their neighborhood survey, trying to determine how much they should charge for a driveway mascot. Remember, this is what they have found from the survey: *They can sell 100 driveway mascots at a cost of $20, and they will sell 10 fewer mascots for each additional $5 they charge.*

9. Write an equation, make a table, and sketch a graph for the number of driveway mascots the twins can sell for each $5 increment, *x*, in the price of the mascot.

10. Write an equation, make a table, and sketch a graph (on the same set of axes) for the price of the driveway mascot for each $5 increment, *x*, in the price.

11. Write an equation, make a table, and sketch a graph for the revenue the twins will collect for each $5 increment in the price of the mascot.

12. The twins estimate that the cost of supplies will be $250 and they would like to make $2000 in profit from selling driveway mascots. Therefore, they will need to collect $2250 in revenue. Write and solve a quadratic equation that represents collecting $2250 in revenue. Be sure to clearly show your strategy for solving this quadratic equation.

3.6 Curbside Rivalry – Teacher Notes

A Solidify Understanding Task

Purpose: In this task students use their techniques for changing the forms of quadratic expressions (i.e., factoring, completing the square to put the quadratic in vertex form, or using the quadratic formula to find the x-intercepts) as strategies for solving quadratic equations.

Core Standards Focus:

A.REI.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Note for Mathematics II A.REI.4a, A.REI.4b

Extend to solving any quadratic equation with real coefficients, including those with complex solutions.

A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Note for Mathematics II A.REI.7

Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

Related Standards: A.SSE.1

Launch (Whole Class):

Before starting this task, it will be helpful to point out some terminology used with quadratics. In previous tasks we have been working with quadratic functions, $f(x) = ax^2 + bx + c$. In this task we will be working with quadratic equations, $ax^2 + bx + c = 0$. We will also refer to quadratic expressions, $ax^2 + bx + c$. Introduce students to the words *roots* and *zeroes* as ways of referring to the *x*-values that are solutions to a quadratic equation or the *x*-values that make a quadratic expression zero. These words can also be used to refer to the *x*-intercepts of a quadratic function that crosses the *x*-axis.

Point out to students that the goal of this task is to learn how to write and solve quadratic equations that arise from different problem situations, and that they will experiment with ways of using the form-changing techniques of previous tasks to support the work of solving quadratic equations.

As part of the launch, read through the context of the task and have students work on question 1 where they will write a simple quadratic equation, $x^2 = 48$ to represent the context. Make sure that students understand they can solve for *x* by taking the square root of both sides of this equation. They developed strategies for working with such radical expressions in task 3.4 *Radical Ideas*. Point out that while there are two numbers we can square to get 48, only the positive square root of 48 makes sense in this context. Also, for the purpose of the context, decimal approximations for square roots provide reasonable solutions.

Also start problem 2 together, before setting students to work on the task. Help students recognize that a quadratic equation that would represent this situation would be $x(x+8) = 48$. Ask students how they might solve such an equation. One method they might suggest would be guess and check. Another method might be to graph the quadratic $y = x(x+8)$ and the line $y = 48$ and look for their points of intersection as Zac did in task 3.4. Point out that the task will help them think about more strategies, particularly algebraic strategies, that they might use on these types of problems.

Explore (Small Group):
After problem 3 the task suggests a typical algebraic strategy that might be used to solve these types of quadratic equations. For example, to solve question 4, multiply out the quadratic expression on the left, and then subtract 48 from both sides to get $x^2 + 8x - 48 = 0$ as an equivalent equation. Solving this equation would be like trying to find the *x*-intercepts of the quadratic function $f(x) = x^2 + 8x - 48$. Ask students how they might find these *x*-intercepts. Try to press for two strategies: finding the factors and determining what values of *x* make the factors zero; or, using the quadratic formula from the previous task *Throwing an Interception.* Similar approaches will work for questions 5-8. Help students see that factoring is an effective strategy sometimes, but not all quadratic expressions factor nicely. The quadratic formula can always be used to find the solutions, but can be cumbersome to apply.

Questions 9-12 provide an opportunity to create and solve a quadratic equation that deals with optimization. Students write two linear equations to represent the number of mascots to be sold, $y = 100 - 10x$, and the price of each mascot, $y = 20 + 5x$. The product of these two functions, $y = (100 - 10x)(20 + 5x)$, represents the revenue collected. A typical question one might ask is to find the maximum revenue, which could be answered by finding the vertex of this function. In this task the question asked—when will the revenue equal $2250—leads to a quadratic equation to be solved: $2250 = (100 - 10x)(20 + 5x)$. Again the strategy of changing the form of this equation to an equivalent quadratic equation where one side equals zero provides a path to a solution. Students may also recognize that the solution shows up in the table for revenue.

Discuss (Whole Class):

Focus the whole class discussion on this concept: Since the solutions to quadratic equations of the form $f(x) = 0$ occur when the function crosses the *x*-axis, setting factors equal to 0 or using the quadratic formula are reasonable strategies for solving such equations. Select problems from the task that seem the most helpful for your students, including at least one problem that can be solved by factoring and one that requires the quadratic formula.

Given time, it would be good to discuss questions 9-12 to remind students that (1) quadratics are the product of two linear functions, (2) the *x*-intercepts of the quadratic function are the *x*-intercepts of the individual linear factors, and (3) the vertex of the quadratic is on the axis of symmetry halfway between the *x*-intercepts. It would be good to connect the graphical, numerical and algebraic ways the solutions to this problem get represented by examining the data in the table of the revenue, by graphing the revenue function and the horizontal line representing the desired revenue, and by solving this equation using the quadratic formula.

Aligned Ready, Set, Go: Solving Quadratic and Other Equations 3.6