5.2 Do You See What I See?

A Develop Understanding Task

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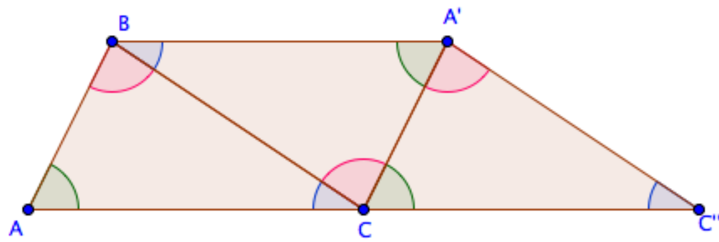
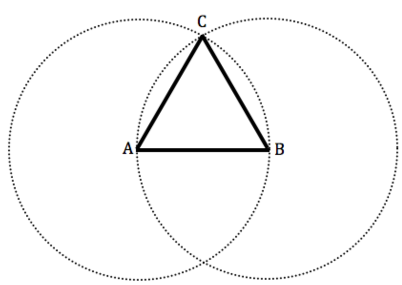
In the previous task, *How Do You Know That*, we saw how the following diagram could be constructed by rotating a triangle about the midpoint of two of its sides. The final diagram suggests that the sum of the three angles of a triangle is 180°. This diagram "tells a story" because you saw how it was constructed through a sequence of steps. You may even have carried out those steps yourself.



Sometimes we are asked to draw a conclusion from a diagram when we are given the last diagram in a sequence steps. We may have to mentally reconstruct the steps that got us to this last diagram, so we can believe in the claim the diagram wants us to see.

1. For example, what can you say about the triangle in the following diagram?

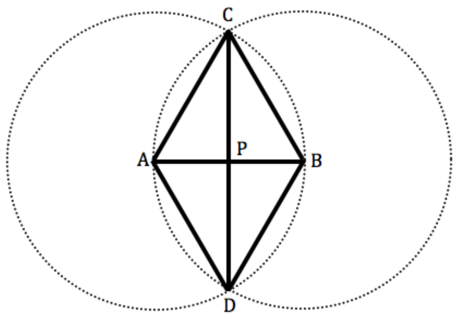


2. What convinces you that you can make this claim? What assumptions, if any, are you making about the other figures in the diagram?

3. What is the sequence of steps that led to this final diagram?

4. What can you say about the triangles, quadrilateral, or diagonals of the quadrilateral that appear in the following diagram? List several conjectures that you believe are true.

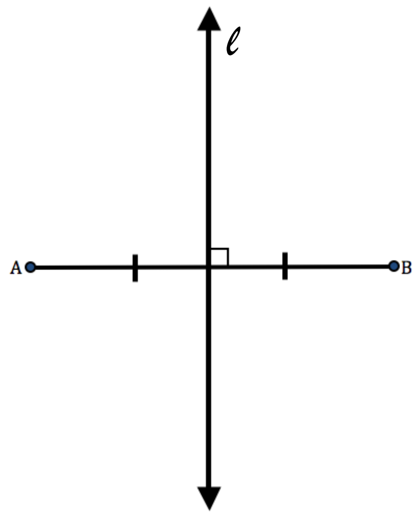
   Given: $\odot A \cong \odot B$

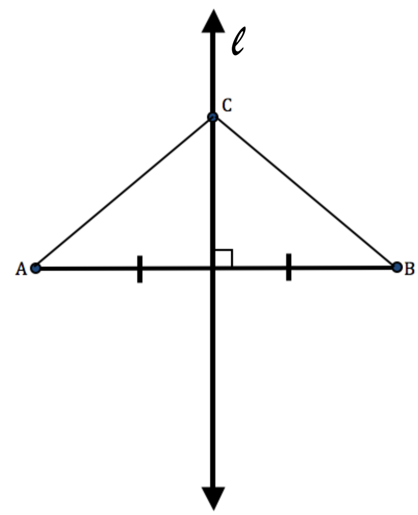


5. Select one of your conjectures and write a paragraph convincing someone else that your conjecture is true. Think about the sequence of statements you need to make to tell your story in a way that someone else can follow the steps and construct the images you want them to see.

6. Now pick a second claim and write a paragraph convincing someone else that this claim is true. You can refer to your previous paragraph, if you think it supports the new story you are trying to tell.

7. Here is one more diagram. Describe the sequence of steps that you think were used to construct this diagram beginning with the figure on the left and ending with the figure on the right.





Travis and Tehani are doing their math homework together. One of the questions asks them to prove the following statement.

> *The points on the perpendicular bisector of a segment are equidistant from the endpoints of the segment*?

Travis and Tehani think the diagram above will be helpful to prove this statement, but they know they will need to say more than just describe how to create this diagram. Travis starts by describing the things they know, and Tehani tries to keep a written record by jotting notes down on a piece of paper.

8. In the table below, record in symbolic notation what Tehani may have written to keep track of Travis' statements.

Tehani's Notes	Travis' Statements
	We need to start with a segment and its perpendicular bisector already drawn.
	We need to show that *any* point on the perpendicular bisector is equidistant from the two endpoints, so I can pick any arbitrary point on the perpendicular bisector. Let's call it C.
	We need to show that this point is the same distance from the two endpoints.
	If we knew the two triangles were congruent, we could say that the point on the perpendicular bisector is the same distance from each endpoint.
	So, what do we know about the two triangles that would let us say that they are congruent?
	We know that both triangles contain a right angle.
	And we know that the perpendicular bisector cuts segment AB into two congruent segments.
	Obviously, the segment from C to the midpoint of segment AB is a side of both triangles.
	So, the triangles are congruent by the SAS triangle congruence criteria.
	Since the triangles are congruent, segments AC and BC are congruent.
	And, that proves that point C is equidistant from the two endpoints!

9. Tehani thinks Travis is brilliant, but she would like the ideas to flow more easily from start to finish. Arrange Tehani's symbolic notes in a way that someone else could follow the argument and see the connections between ideas.

10. Would your justification be true regardless of where point C is chosen on the perpendicular bisector? Why?