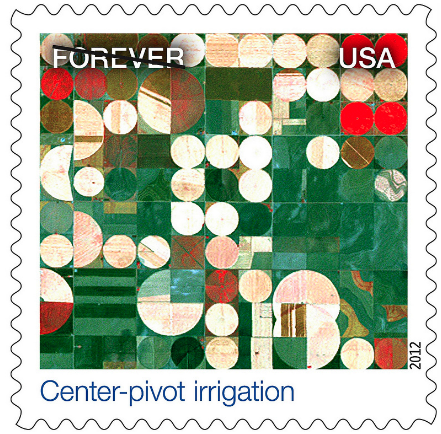
5.8 Centers of a Triangle
A Practice Understanding Task



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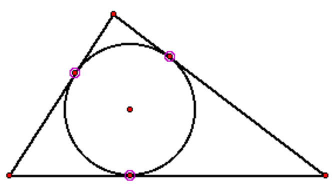
Kolton, Kevin and Kara have been asked by their fathers to help them solve some interesting geometry problems.

Problem 1

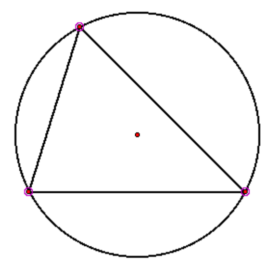
Kolton's father installs sprinkling systems for farmers. The systems he installs are called "Center Pivot Irrigation Systems" since the sprinklers are on a long pipe that rotates on wheels around a center point, watering a circular region of crops. You may have seen such "crop circles" from an airplane.



Sometimes Kolton's father has to install sprinkler systems on triangular shaped pieces of land. He wants to be able to locate the "pivot point" in the triangular field so the circle being watered will touch each of the three fences that form the boundaries of the field. He has asked for Kolton's help with this problem, since Kolton is currently studying geometry in high school.

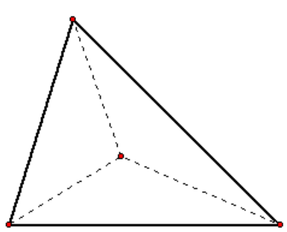
Problem 2

Kara's father installs cell towers. Since phone signals bounce from tower to tower, they have to be carefully located. Sometimes Kara's father needs to locate a new tower so that it is equidistant from three existing towers. He thinks of the three towers that are already in place as the vertices of a triangle, and he needs to be able to find a point in this triangle where he might locate the new tower so that it is equidistant from the other three. He has asked Kara to help him with this problem since she is also studying geometry in school.



Problem 3

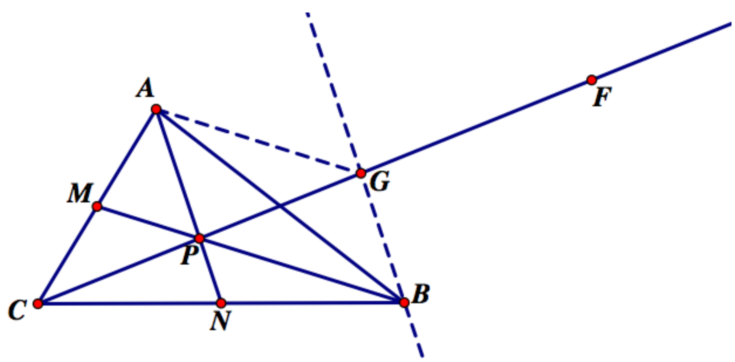
Kevin's father is an artist and has been commissioned by the city to build an art project in the park. His proposal consists of several large pyramids with different shaped triangles balanced on the vertex points of the pyramids. Kevin's father needs to be able to find the point inside of a triangle that he calls "the balancing point." He has asked Kevin to use his knowledge of geometry to help him solve this problem.



Kolton, Kevin and Kara's geometry teacher has suggested they try locating points in the interior of triangles where medians, altitudes, angle bisectors, or perpendicular bisectors of the sides intersect.

1. Try out the experiment suggested by the students' geometry teacher. Which set of line segments seem to locate a point in the triangle that best meets the needs of each of their fathers?

2. Kolton, Kevin and Kara have noticed something interesting about these sets of line segments. To their surprise, they notice that all three medians of a triangle intersect at a common point. Likewise, the three altitudes also intersect at a common point. So do the three angle bisectors, and the three bisectors of the sides. They think their fathers will find this interesting, but they want to make sure these observations are true for all triangles, not just for the ones they have been experimenting on. The diagrams and notes below suggest how each is thinking about the proof they want to show his or her father. Use these notes and diagrams to write a convincing proof.

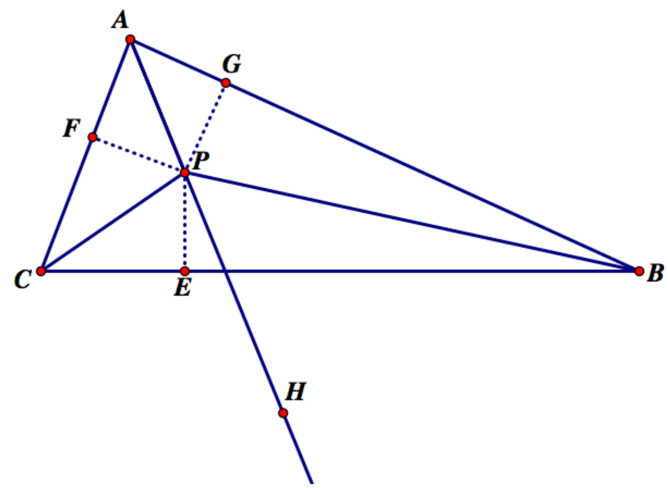


Kevin's Notes

What I did to create this diagram:

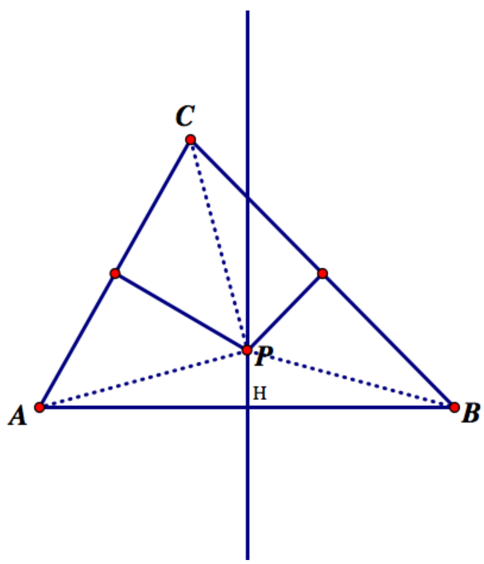
Point *M* is the midpoint of side AC, and point *N* is the midpoint of side CB. Therefore, $\overline{AN}$ and $\overline{BM}$ are medians of the triangle. I then drew ray *CF* through point *P*, the intersection of the two medians. My question is, "Does this ray contain the third median?" So, I need to find a way to answer that question. As I was thinking about this, I thought I could visualize a parallelogram with its diagonals, so I drew line *GB* to be parallel to median *AN*, and then connected vertex *A* to point *G* on the ray. Quadrilateral *AGBP* looks like a parallelogram, but I'm not so sure. And I am wondering if that will help me with my question about the third median. What do you think?

Kolton's Notes



What I did to create this diagram:

I constructed the angle bisectors of angle *ACB* and angle *ABC.* They intersected at point *P*. So I wouldn't get confused by so many lines in my diagram, I erased the rays that formed the angle bisectors past their point of intersection. I then drew ray *AH* through point *P*, the point of intersection of the two angle bisectors. My question is, "Does this ray bisect angle *CAB*?" While I was thinking about this question, I noticed that I had created three smaller triangles in the interior of the original triangle. I constructed the altitudes of these three triangles (they are drawn as dotted lines). When I added the dotted lines, I started seeing kites in my picture. I'm wondering if thinking about the smaller triangles or the kites might help me prove that ray *AH* bisects angle *CAB.*



Kara's Notes

What I did to create this diagram:

I constructed the perpendicular bisectors of side *AC* and side *BC.* They intersected at point *P*. So I wouldn't get confused by so many lines in my diagram, I erased the rays that formed the perpendicular bisectors past their point of intersection. I then constructed a line perpendicular to side *AB* through point *P*, the point of intersection of the two perpendicular bisectors. I named the point where this line intersected side *AB* point *H.* My question is, "Does this perpendicular line also bisect side *AB*?" While I was thinking about this question, I noticed that I had creates some quadrilaterals in the interior of the original triangle. Since quadrilaterals in general don't have a lot of interesting properties, I decided to make some triangles by dotting in line segments drawn from *P* to each of the vertices of the original triangle. I'm wondering if thinking about these smaller triangles might help me prove that line *PH* bisects side *AB.*