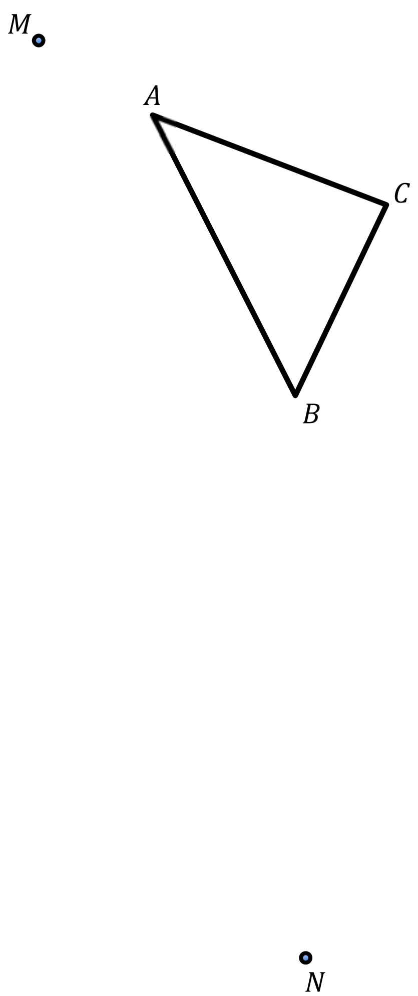
6.2 Triangle Dilations

A Solidify Understanding Task

1. Given ΔABC, use point M as the center of a dilation to locate the vertices of a triangle that has side lengths that are three times longer than the sides of ΔABC.

2. Now use point N as the center of a dilation to locate the vertices of a triangle that has side lengths that are one-half the length of the sides of ΔABC.



3. Label the vertices in the two triangles you created in the diagram above. Based on this diagram, write several proportionality statements you believe are true. First write your proportionality statements using the names of the sides of the triangles in your ratios. Then verify that the proportions are true by replacing the side names with their measurements, measured to the nearest millimeter.

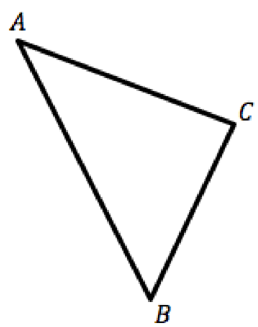
   My list of proportions: (try to find at least 10 proportionality statements you believe are true)

4. Based on your work above, under what conditions are the corresponding line segments in an image and its pre-image parallel after a dilation? That is, which word best completes this statement?

   After a dilation, corresponding line segments in an image and its pre-image are [never, sometimes, always] parallel.

5. Give reasons for your answer. If you choose "sometimes", be very clear in your explanation how to tell when the corresponding line segments before and after the dilation are parallel and when they are not.

Given ΔABC, use point A as the center of a dilation to locate the vertices of a triangle that has side lengths that are twice as long as the sides of ΔABC.



6. Explain how the diagram you created above can be used to prove the following theorem:

   The segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.

6.2 Triangle Dilations – Teacher Notes

A Solidify Understanding Task

Purpose: One purpose of this task is to solidify and formalize the definition of dilation: A dilation is a transformation of the plane, such that if O is the center of the dilation and a non-zero number k is the scale factor, then P' is the image of point P if O, P and P' are collinear and $\frac{OP'}{OP} = k$.

A second purpose of this task is to examine proportionality relationships between sides of similar figures by identifying and writing proportionality statements based on corresponding sides of the similar figures.

A third purpose is to examine a similarity theorem that can be proved using dilation: *a line parallel to one side of a triangle divides the other two proportionally.*

Core Standards Focus:

G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally.

Related Standards: G.SRT.1, 8.G.4

Launch (Whole Class): [questions 1-3]
Remind students that in previous math classes they have studied proportionality relationships. Discuss what it means to say that quantities are proportional and review how to write proportionality relationships symbolically. In Math 8 students learned that dilations produce similar figures and remind them of the definition they used for similar figures: (CCSS-M 8.G.4) *Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations*. Point out to students that questions 1 and 2 ask them to create some similar figures using dilations, and that question 3 will ask them to write some proportionality statements based on these similar figures.

Explore (Small Group): [questions 1-3]
As students identify the vertices of the two dilated triangles on question 1 and 2 encourage them to label corresponding vertices as A', B', C' and A'', B'', C'' so they can refer to the sides of the triangles by name when writing proportions for question 3. Verify that they are correctly locating the vertices of the two triangles with scale factor 3 centered at M and scale factor ½ centered at N.

Observe the proportions that students write for question 3, and try to identify (and press for) a variety of ways of writing these proportions. For example, in one type of proportion the ratios would consist of comparing corresponding sides from each of two triangles $\left(\frac{AB}{A'B'}=\frac{BC}{B'C'}\right)$; in another type of proportion the ratios would consist of two sides taken from one triangle compared to the same two sides of another triangle $\left(\frac{AB}{BC}=\frac{A'B'}{B'C'}\right)$. Watch for students who write proportions between the sides of the largest triangle and the smallest triangle, recognizing that these triangles are also similar to each other.

Discuss (Whole Class): [questions 1-3]
Begin the discussion by posing this formal definition of dilation: *A dilation is a transformation of the plane, such that if O is the center of the dilation and a non-zero number k is the scale factor, then P' is the image of point P if O, P and P' are collinear and* $\frac{OP'}{OP}=k$, and ask students how this definition showed up in their work with questions 1 and 2. Have students record this definition in their notes or on a class poster.

Select several students to share proportions they wrote for question 3, making sure that a variety of ways of identifying proportionality statements are discussed, as outlined in the explore above.

End this discussion by reviewing the definition of similarity (*a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations*) and ask students to verify that the largest and smallest triangles are similar by describing a sequence of transformations that exhibits the similarity between them. (One possibility is to translate the smallest triangle to the largest triangle so that vertex *A"* corresponds with vertex *A'*, then dilate the small triangle with the dilation centered at *A"* to superimpose it on top of the largest triangle. Ask students to identify the scale factor of this dilation.)

Launch (Whole Class): [questions 4-6]
The last part of the previous discussion will prepare students to work on questions 4-6. Remind them that in the task *Parallelism Preserved and Protected* they identified some parallel postulates for the rigid motion transformations: translation, rotation and reflection. In question 4 and 5 they will propose a parallel postulate for dilation. They will also have an opportunity to create a diagram and write a proof about a similarity theorem in question 6.

Explore (Small Group): [questions 4-6]
Encourage students to examine the three similar triangles they created by dilation on the previous part of the task. Ask, "What relationships do you notice between corresponding line segments after these dilations? Do you think this will always be the case? Why do you think so?"

On question 6 verify that students have created the diagram correctly, and that they are using features of the diagram to guide their thinking on the proof. Help them focus on using the sides of the triangle as transversals for the third sides of the triangles. Students should notice that since dilations preserve angle measure, in their diagram pairs of corresponding angles are congruent relative to the sides being used as transversals. This proves that the third sides of the triangles are parallel. Since *C* and *B* are midpoints of the larger triangle's sides, the scale factor of the dilation is 2 (going from the smaller triangle to the larger) or ½ (going from the larger triangle to the smaller).

Discuss (Whole Class): [questions 4-6]
Start the discussion by having students state their parallel postulate for dilations as requested in questions 4 and 5. Note that in question 6 they have actually proved this postulate for one type of dilation: a triangle dilated about one of its vertices. Based on our experiments in questions 1 and 2 we are going to accept this statement as a postulate for our approach to transformational geometry, as suggested by the common core standards.

Have a student share his proof for question 6. This is a special case of a more general theorem that will be explored in detail in *6.4 Cut By a Transversal.* Have students record it in their notes or on a classroom poster.

Aligned Ready, Set, Go: Similarity & Right Triangle Trigonometry 6.2