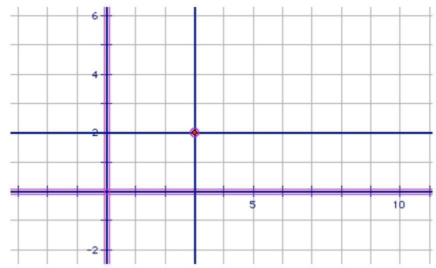
Is It Right?

A Solidify Understanding Task

In *Leaping Lizards* you probably thought a lot about perpendicular lines, particularly when rotating the lizard about a 90° angle or reflecting the lizard across a line.

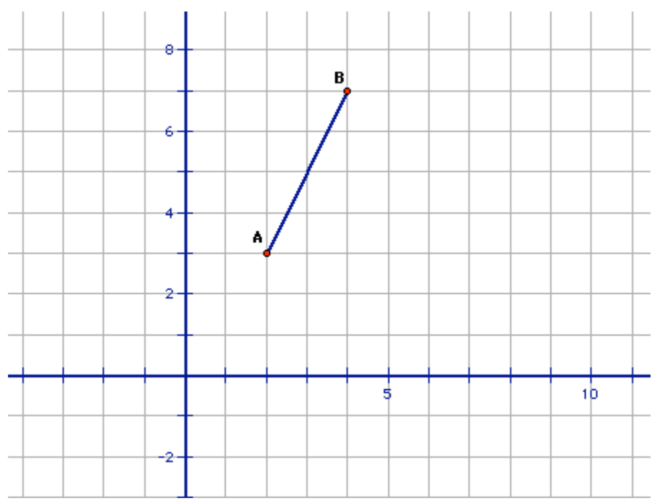
In previous tasks, we have made the observation that *parallel lines have the same slope*. In this task we will make observations about the slopes of perpendicular lines. Perhaps in *Leaping Lizards* you used a protractor or some other tool or strategy to help you make a right angle. In this task we consider how to create a right angle by attending to slopes on the coordinate grid.

We begin by stating a fundamental idea for our work: *Horizontal and vertical lines are perpendicular.* For example, on a coordinate grid, the horizontal line $y = 2$ and the vertical line $x = 3$ intersect to form four right angles.



But what if a line or line segment is not horizontal or vertical?
How do we determine the slope of a line or line segment that will be perpendicular to it?

<u>Experiment 1</u>



1. Consider the points A (2, 3) and B (4, 7) and the line segment, $\overline{AB}$, between them. What is the slope of this line segment?

2. Locate a third point C (x, y) on the coordinate grid, so the points A (2, 3), B (4, 7) and C (x, y) form the vertices of a right triangle, with $\overline{AB}$ as its hypotenuse.

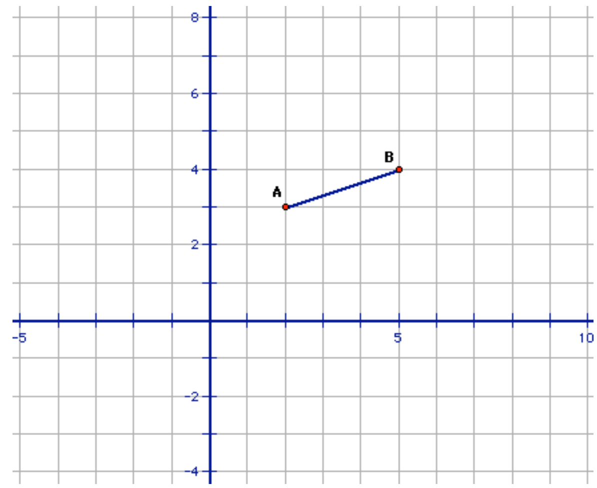
3. Explain how you know that the triangle you formed contains a right angle?

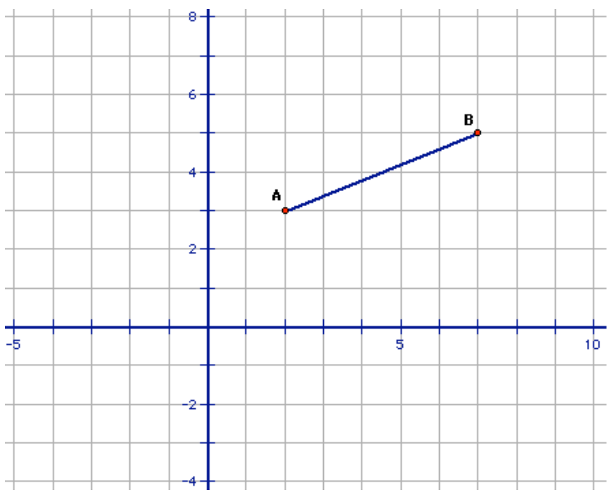
4. Now rotate this right triangle 90° about the vertex point (2, 3). Explain how you know that you have rotated the triangle 90°.

5. Compare the slope of the hypotenuse of this rotated right triangle with the slope of the hypotenuse of the pre-image. What do you notice?

Experiment 2

Repeat steps 1-5 above for the points A (2, 3) and B (5, 4).



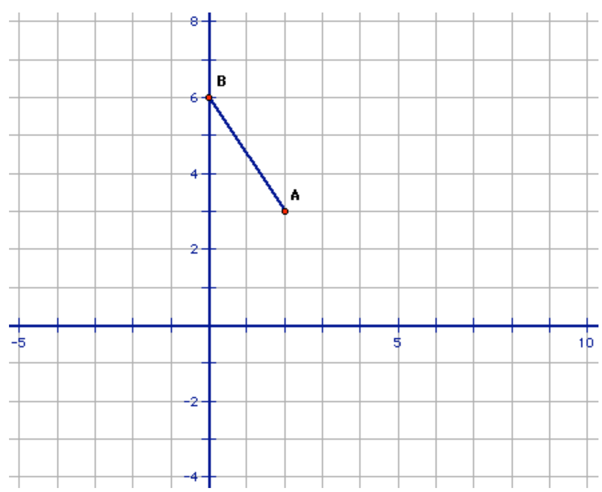


Experiment 3

Repeat steps 1-5 above for the points A (2, 3) and B (7, 5).

Experiment 4

Repeat steps 1-5 above for the points A (2, 3) and B (0, 6).



Based on experiments 1-4, state an observation about the slopes of perpendicular lines.

While this observation is based on a few specific examples, can you create an argument or justification for why this is always true? (Note: You will examine a formal proof of this observation in the next module.)

Is It Right? – Teacher Notes

Purpose: In this task students make a conjecture about the slopes of perpendicular lines. This observation will be formally proved in a later task, but the representation-based argument presented in this task suggests that this is a reasonable generalization across all cases. In the previous task, students encountered the idea of perpendicularity when they were asked to rotate the lizard 90°, and they may also have noticed the idea when they considered the relationship between the line of reflection and the points on the reflected image of the lizard. Students might have used the square corner of a piece of paper or a protractor to measure these right angles. In this task they consider how the coordinate grid can be used to determine if two lines are perpendicular. This is one example of the connections that can be made between coordinate geometry and transformations. It is powerful for students to be able to draw upon two different representational systems to think about the same ideas.

In this task, students also consider a definition of perpendicular that is related to reflections: *Two lines are perpendicular if they meet to form congruent adjacent angles*.

Core Standards Focus:

G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Related Standards: G.GPE.4

Launch (Whole Class):
Students already have a sense of what it means for two lines to be perpendicular. Have them state their "definitions" and list their ideas on the board (e.g., they meet at right angles, they form 90° angles, if one is horizontal then the other is vertical, etc.) Then propose the following definition of perpendicular lines: *Lines are perpendicular if they meet to form congruent adjacent angles*. Ask students how this definition connects to the ideas they have recorded on the board about perpendicular lines (e.g., since the two adjacent angles form a straight angle measuring 180°, if the adjacent angles are also congruent they must each measure 90°; by definition, a right angle measures 90°). Ask students how they might justify this definition of perpendicular lines using transformations. Students might suggest folding or reflecting half of the line onto the other half, creasing the fold along the perpendicular line or using the perpendicular line as the line of

reflection. Since reflections preserve angle measure, the image angle and its adjacent pre-image angle are congruent.

With these ideas about perpendicular lines activated for students, turn their attention to the experiments outlined in the task. Inform students that they are going to make a conjecture about lines that are perpendicular on a coordinate grid. For the sake of these experiments, we will agree that horizontal and vertical lines are perpendicular.

As part of the launch, make sure that students can accurately plot point C to form the third vertex of a right triangle ABC with segment AB as the hypotenuse. There are two positions where C can be located so that the legs of the right triangle lie along horizontal and vertical lines, thus guaranteeing that we have a right angle at C.

Explore (Small Group):
The exploration asks students to rotate a right triangle 90° around a vertex located at one of the acute angles. Since the legs of the right triangle are oriented along horizontal and vertical lines, students will know that they have rotated the triangle 90° when the leg adjacent to point A in the pre-image right triangle is oriented in the other direction—horizontal or vertical—in the resulting image right triangle. (See diagrams below)

Watch for students to plot point C in a correct location, and listen for their arguments as to how they know they have rotated the right triangle 90°. Ask students how they know that the hypotenuse of the rotated right triangle is perpendicular to the hypotenuse of the original right triangle as a result of this rotation. It may be helpful to have students use a compass to connect points B and C with their image points along circles, centered at A—particularly if this was a strategy used to analyze the rotation in the previous task.

Make sure that students are recording the slopes of the hypotenuse of the pre-image right triangle and its resulting image right triangle correctly.

Discuss (Whole Class):
Post samples of student work that illustrate what happens when C is located below B and to the right of A, as well as what happens when C is located above A and to the left of B, as in the following diagram:

Have students articulate how they know when they have rotated the triangle 90° about point A by having them identify the horizontal/vertical relationship between leg AC and its image. Have them justify why this means that the hypotenuse of the rotated right triangle is also perpendicular to the hypotenuse of the original right triangle. Then have them state a conjecture about the slope of perpendicular lines based on these four examples. Remind students that four examples do not prove a conjecture, and have them suggest how they might generalize this work—that is, does the visual representation of the perpendicular hypotenuses hold, regardless of the size of the right triangle or the measures of the acute angles in the right triangle? This is an informal argument for an idea that will be formally proven in a task in the next module.

Aligned Ready, Set, Go: Congruence, Construction and Proof 2