3.1 The In-Betweeners

A Develop Understanding Task

Now that you've seen that there are contexts for continuous exponential functions, it's a good idea to start thinking about the numbers that fill in between the values like 2^2 and 2^3 in an exponential function. These numbers are actually pretty interesting, so we're going to do some exploring in this task to see what we can find out about these "in-betweeners".

Let's begin in a familiar place:

1. Complete the following table.

x	0	1	2	3	4
$f(x) = 4 \cdot 2^x$	4				

2. Plot these points on the graph at the end of this task, and sketch the graph of $f(x)$.

Let's say we want to create a table with more entries, maybe with a point halfway between each of the points in the table above. There are a couple of ways that we might think about it. We'll begin by letting our friend Travis explain his method.

Travis makes the following claim:

"If the function doubles each time x goes up by 1, then half that growth occurs between 0 and ½ and the other half occurs between ½ and 1. So for example, we can find the output at x = ½ by finding the average of the outputs at x = 0 and x = 1."

3. Fill in the parts of the table below that you've already computed, and then decide how you might use Travis' strategy to fill in the missing data. Also plot Travis' data on the graph at the end of the task.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	4								

4. Comment on Travis' idea. How does it compare to the table generated in problem 1? For what kind of function would this reasoning work?

Miriam suggests they should fill in the data in the table in the following way:

"I noticed that the function increases by the same factor each time x goes up 1, and I think this is like what we did in *Geometric Meanies*. It seems like this property should hold over each half-interval as well."

5. Fill in the parts of the table below that you've already computed in problem 1, and then decide how you might use Miriam's new strategy to fill in the missing data. As in the table in problem 1, each entry should be multiplied by some constant factor to get the next entry, and that factor should produce the same results as those already in recorded in the table. Use this constant factor to complete the table. Also plot Miriam's data on the graph at the end of this task.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	4								

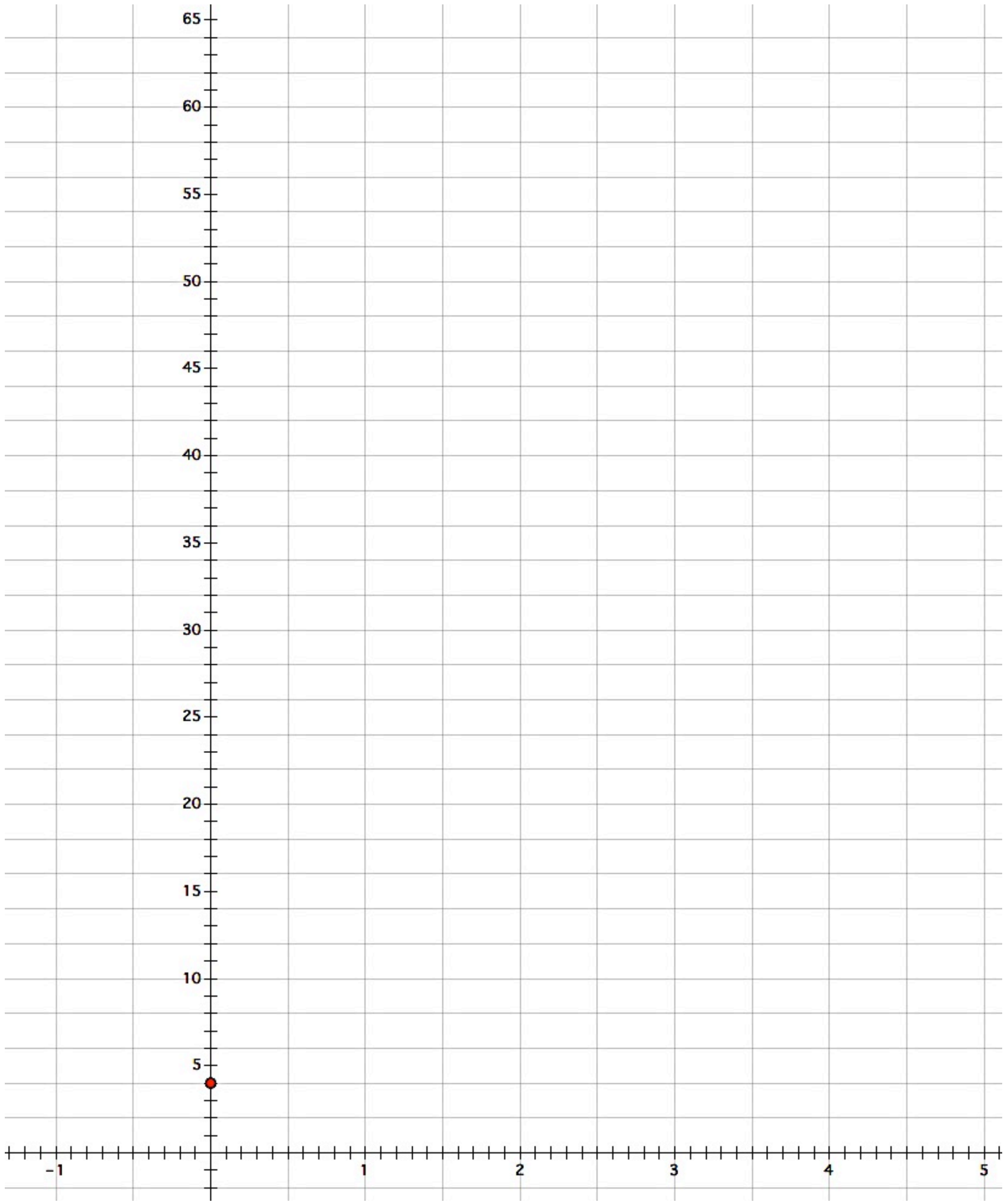
6. What if Miriam wanted to find values for the function every third of the interval instead of every half? What constant factor would she use for every third of an interval to be consistent with the function doubling as x increases by 1. Use this multiplier to complete the following table.

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
$f(x)$	4									

7. What number did you use as a constant factor to complete the table in problem 4?

8. What number did you use as a constant factor to complete the table in problem 5?

9. Give a detailed description of how you would estimate the output value $f(x)$, for $x = 5/3$.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