5.3 It's All In Your Head

A Solidify Understanding Task

In the previous task you were asked to justify some claims by writing paragraphs explaining how various figures were constructed and how those constructions convinced you that the claims were true. Perhaps you found it difficult to say everything you felt you just knew.

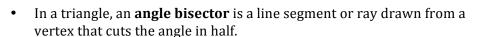


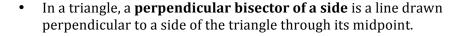
Sometimes we all find it difficult to explain our ideas and to get those ideas out of our heads and written down or paper.

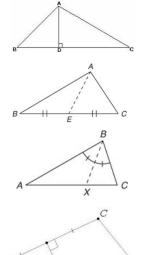
Organizing ideas and breaking complex relationships down into smaller chunks can make the task of proving a claim more manageable. One way to do this is to use a flow diagram.

First, some definitions:

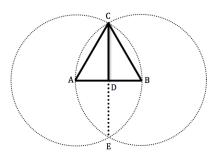
- In a triangle, an **altitude** is a line segment drawn from a vertex perpendicular to the opposite side (or an extension of the opposite side).
- In a triangle, a median is a line segment drawn from a vertex to the midpoint of the opposite side.







Travis used a compass and straightedge to construct an equilateral triangle. He then folded his diagram across the two points of intersection of the circles to construct a line of reflection. Travis, Tehani, Carlos and Clarita are trying to decide what to name the line segment from C to D.

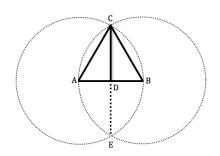


Travis thinks the line segment they have constructed is also a median of the equilateral triangle. Tehani thinks it is an angle bisector. Clarita thinks it is an altitude and Carlos thinks it is a perpendicular bisector of the opposite side. The four friends are trying to convince each other that they are right.

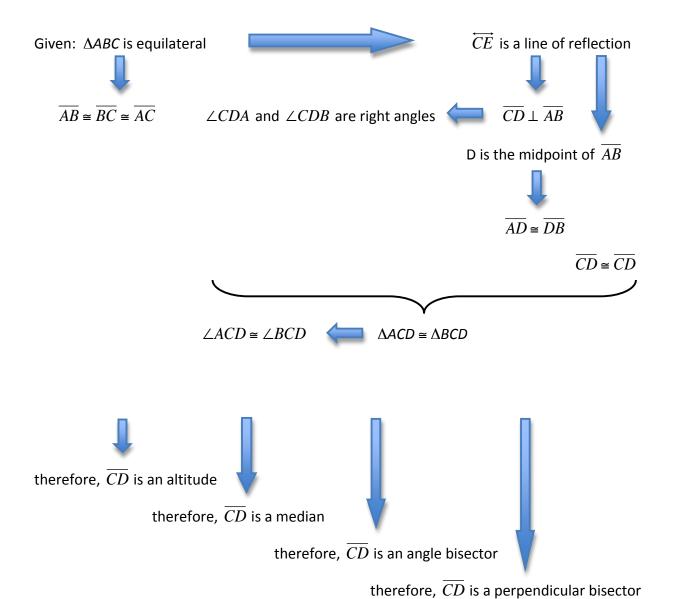
© 2013 Mathematics Vision Project | M ${f V}$ P



Here is a flow diagram of statements that can be written to describe relationships in the diagram, or conclusions that can be made by connecting multiple ideas.



Use four different colors to identify the statements each of the students—Travis, Tehani, Carlos and Clarita might use to make their case.





Match each of the arrows and braces in the flow diagram with one of the following reasons that justifies why you can make the connection between the statement (or statements) previously accepted as true and the conclusion that follows:

- 1. Definition of reflection
- 2. Definition of translation
- 3. Definition of rotation
- 4. Definition of an equilateral triangle
- 5. Definition of perpendicular
- 6. Definition of midpoint
- 7. Definition of altitude
- 8. Definition of median
- 9. Definition of angle bisector
- 10. Definition of perpendicular bisector
- 11. Equilateral triangles can be folded onto themselves about a line of reflection
- 12. Equilateral triangles can be rotated 60° onto themselves
- 13. SSS triangle congruence criteria
- 14. SAS triangle congruence criteria
- 15. ASA triangle congruence criteria
- 16. Corresponding parts of congruent triangles are congruent

Travis and his friends have seen their teacher write two-column proofs in which the reasons justifying a statement are written next to the statement being made. Travis decides to turn his argument into a two column proof, as follows.

Statements	Reasons
ΔABC is equilateral	Given
\overrightarrow{CE} is a line of reflection	Equilateral triangles can be folded onto themselves about a line of reflection
D is the midpoint of \overline{AB}	Definition of reflection
\overline{CD} is a median	Definition of median

Write each of Tehani's, Carlos', and Clarita's arguments in two-column proof format.

