6.5 Symmetries of Quadrilaterals

A Develop Understanding Task

A line that reflects a figure onto itself is called a **line of symmetry**. A figure that can be carried onto itself by a rotation is said to have **rotational symmetry**.

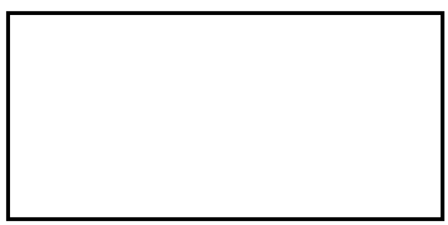
Every four-sided polygon is a **quadrilateral**. Some quadrilaterals have additional properties and are given special names like squares, parallelograms and rhombuses. A **diagonal** of a quadrilateral is formed when opposite vertices are connected by a line segment. In this task you will use rigid-motion transformations to explore line symmetry and rotational symmetry in various types of quadrilaterals.

1. A **rectangle** is a quadrilateral that contains four right angles. Is it possible to reflect or rotate a rectangle onto itself?

For the rectangle shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the rectangle onto itself.



Describe the rotations and/or reflections that carry a rectangle onto itself. (Be as specific as possible in your descriptions.)

2. A **parallelogram** is a quadrilateral in which opposite sides are parallel. Is it possible to reflect or rotate a parallelogram onto itself?

For the parallelogram shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the parallelogram onto itself.

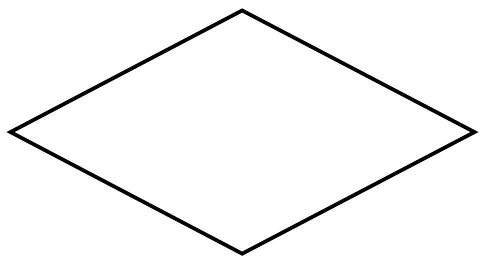
Describe the rotations and/or reflections that carry a parallelogram onto itself. (Be as specific as possible in your descriptions.)

3. A **rhombus** is a quadrilateral in which all sides are congruent. Is it possible to reflect or rotate a rhombus onto itself?

For the rhombus shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the rhombus onto itself.



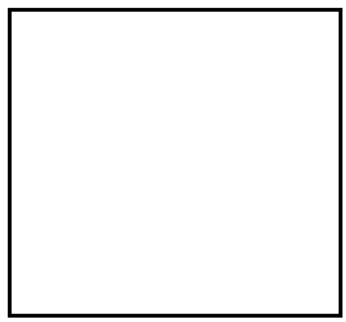
Describe the rotations and/or reflections that carry a rhombus onto itself. (Be as specific as possible in your descriptions.)

4. A **square** is both a rectangle and a rhombus. Is it possible to reflect or rotate a square onto itself?

For the square shown below, find

- any lines of reflection, or
- any centers and angles of rotation

that will carry the square onto itself.



Describe the rotations and/or reflections that carry a square onto itself. (Be as specific as possible in your descriptions.)

5. A **trapezoid** is a quadrilateral with one pair of opposite sides parallel. Is it possible to reflect or rotate a trapezoid onto itself?

Draw a trapezoid based on this definition. Then see if you can find

- any lines of symmetry, or
- any centers of rotational symmetry

that will carry the trapezoid you drew onto itself.

If you were unable to find a line of symmetry or a center of rotational symmetry for your trapezoid, see if you can sketch a different trapezoid that might possess some type of symmetry.

6.5 Symmetries of Quadrilaterals – Teacher Notes

A Develop Understanding Task

Purpose: In this learning cycle, students focus on classes of geometric figures that can be carried onto themselves by a transformation—figures that possess a line of symmetry or rotational symmetry. In this task the idea of "symmetry" is surfaced relative to finding lines that reflect a figure onto itself, or determining if a figure has rotational symmetry by finding a center of rotation about which a figure can be rotated onto itself. This work is intended to be experimental (e.g., folding paper, using transparencies, using technology, measuring with ruler and protractor, etc.), with the definitions of reflection and rotation being called upon to support students' claims that a figure possesses some type of symmetry. The particular classes of geometric figures considered in this task are various types of quadrilaterals.

Core Standards Focus:

G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.

Related Standards: G.CO.4, G.CO.5

Launch (Whole Class):
Discuss the concept of symmetry in terms of finding a rigid-motion transformation that carries a geometric figure onto itself. Help students recognize that two such types of symmetries exist: a line of symmetry might exist that reflects a figure onto itself, or a center of rotation might exist about which a figure might be rotated onto itself. Also remind students of the definitions of "quadrilateral" and "diagonal" giving in the introduction of the task.

Students are to experiment with the various types of quadrilaterals listed in the task to determine if they can find any lines of symmetry or centers and angles of rotation that will carry the given quadrilateral onto itself. You will need to decide what tools to make available for this investigation. For example, you could provide cut-outs of each of the figures which would allow students to find lines of symmetry by folding the figures onto themselves (note that a handout of the set of figures is provided at the end of the teacher notes). This would also be a good task to support using dynamic geometry software programs, such as Geometer's Sketchpad or Geogebra. If you use technology, students will need to be provided with a set of well-constructed quadrilaterals, so they can focus on searching for lines of symmetry and centers of rotation, rather than on the construction of the geometric figures themselves.

Explore (Small Group):
Since students are dealing with classes of quadrilaterals, rather than individual quadrilaterals, in addition to finding the line of symmetry or the center and angle of a rotation, they should also provide some type of justification as to how they know that this symmetry exists for all members of the class. The given definitions for each quadrilateral should support making such an argument.