$\qquad$ Period: $\qquad$

## Advanced Math Summer Work 2014

## Part 1

For this review sheet it is important to show your work ON EVERY PROBLEM.


Step 1


Step 2


Step 3

1. How many squares would there be in Step 50 ?
2. Write an expression for how many squares there would be in Step $n$.
3. How is each part of your expression in Question 1 seen in the figures shown above?
4. Are the following expressions equivalent? Justify your answer.

39n-13
13

$$
3 n+13
$$

5. Solve the following equation. Make sure to justify each step with the properties you use. (i.e. Addition Property of Equality or Division Property of Equality)

$$
7 x+3=59
$$

Justification
$\qquad$
6. What is a solution and what does the solution to \#5 represent?
7. How many solutions are there for the equation $3 x+4=2 x-10$ ?
8. How many solutions are there for the equation $y=\frac{3}{2} x-4$ ? Show how the solution(s) would be represented.
9. Graph the following equation:

$$
y=\frac{-3}{4} x-2
$$


10. A line is drawn below. Write an equation of the line.

11. The slope of a line is $-\frac{3}{2}$ and a point on the line is $(1,2)$. Draw the line and write the equation of the line.

12. Solve the following inequality: $3(x+1) \geq 12 x+21$. Write your answer as an inequality.
13. Graph the inequality from Question \#12 on the line below.
14. Solve the following equation for $\mathrm{X}: \frac{A X-Y}{F}+T=W$
$\qquad$
$\qquad$

## Part 2

1. What strategies have we developed to solve systems of equations?
2. What strategies have we developed to solve systems of inequalities?
3. What does the solution set to a system of an equations look like?
4. What does the solution set to a system of inequalities look like?
5. How many solutions are there to a system of equations? Give a graphical example of each possibility.
6. How many solutions are there to a system of inequalities? Give a graphical example.
7. Solve the following system of equations using substitution and by graphing: $\{x+y=16$ $\left\{\begin{array}{l}x+y=9\end{array}\right.$

Show work for substitution below:

8. Solve the following system of equations using elimination and by graphing:
$\{3 x+5 y=7$
$\left\{\begin{array}{l}2 x-3 y=11\end{array}\right.$

Show work for elimination below

9. Solve the following system of equations using a matrix:
$\{2 x+6 y=18$
$\{3 x+2 y=13$
10. Solve the following system of inequalities:
$\left\{\begin{array}{l}y<3 x-1\end{array}\right.$ $\left\{\begin{array}{l}y \geq-2 x+4\end{array}\right.$


Solve each of the systems of equations below using an appropriate method. Use a matrix on one of the two systems, whichever one the matrix row reduction method fits with best.
11. $\left\{\begin{array}{l}y=-x+2 \\ y=3 x-6\end{array}\right.$
12. $\left\{\begin{array}{l}3 x+2 y=-4 \\ 2 x-2 y=-6\end{array}\right.$

Solve the following systems of inequalities.
13. $\left\{\begin{array}{l}y \leq \frac{3}{4} x-5 \\ y>-2 x+1\end{array}\right.$
14. $\left\{\begin{array}{l}4 x+3 y \leq 24 \\ 6 x+9 y \leq 18\end{array}\right.$

Circle the points that are solutions to the system of inequalities.
15. $\left\{\begin{array}{c}x+y>4 \\ 2 x+3 y \leq 12\end{array}\right.$
16. $\left\{\begin{array}{l}y \leq \frac{1}{2} x-3 \\ y \leq 4 x-3\end{array}\right.$
a. $(0,4)$
a. $(-2,2)$
b. $(4,1)$
b. $(2,1)$
c. $(2,1)$
c. $(0,-3)$

Circle the points that are solutions to the system of equations.
17. $\left\{\begin{array}{l}y=\frac{1}{2} x-3 \\ y=4 x-3\end{array}\right.$
18. $\left\{\begin{array}{c}y=3 x+7 \\ y=-3 x-5\end{array}\right.$
a. $(0,3)$
a. $(0,0)$
b. $(10,2)$
b. $(-2,1)$
c. No solution
c. $(-1,4)$
19. Write the system of inequalities that matches the following graph

20. When graphing an inequality what does a dotted line mean?

Solve the following systems of equations by using a method other than graphing. Use whatever method is most efficient for the given system.

Write your answer as a coordinate point.
21. $\left\{\begin{array}{c}x=y-1 \\ -3 x+2 y=-1\end{array}\right.$
22. $\left\{\begin{array}{c}-7 x-2 y=-13 \\ x-2 y=11\end{array}\right.$
23. You are shopping at Walmart for popsicles. You want to get blue-raspberry and cherry flavors. The blue-raspberry are bigger, so they cost $\$ 1.50$ each while the cherry are only $\$ 1$. Walmart is having a special and you get a free gift if you spend over $\$ 25$. You want to find all of the different combinations of popsicles that you could buy and get a free gift.
(a) Write an inequality for the situation above.
(b) Find all of the solutions to your inequality
(c) Are all of the solutions that you found in (b) viable (Do they all make sense for the story problem)?

## Part 3

Classify as each function the following two ways:
(a) Arithmetic/Geometric
(b) Recursive/Explicit Function

1. $h(x)=3 x+1$
2. $m(t)=m(t-1)+4$
3. $f(x)=3 \cdot f(x-1)$
4. $r(x)=4(x-1)+1$
5. $f(t)=4 \cdot 3^{t}$

Write Recursive Functions for the following tables:
6.

| $x$ | $f(x)$ |
| :---: | :---: |
| 0 | 4 |
| 1 | 11 |
| 2 | 18 |
| 3 | 25 |

7. 

| x | $\mathrm{f}(\mathrm{x})$ |
| :---: | :---: |
| 1 | 4 |
| 2 | 12 |
| 3 | 36 |
| 4 | 108 |

8. 

| $x$ | $f(x)$ |
| :---: | :---: |
| 3 | 12 |
| 4 | 24 |
| 5 | 48 |
| 6 | 96 |

Write Explicit Functions for the following tables:
9.

| $x$ | $f(x)$ |
| :---: | :---: |
| 0 | 4 |
| 1 | 11 |
| 2 | 18 |
| 3 | 25 |

10. 

| $x$ | $f(x)$ |
| :---: | :---: |
| 1 | 4 |
| 2 | 12 |
| 3 | 36 |
| 4 | 108 |

11. 

| $x$ | $f(x)$ |
| :---: | :---: |
| 3 | 12 |
| 4 | 24 |
| 5 | 48 |
| 6 | 96 |

Create a table for each of the following functions:
12. $y(x)=-2 \cdot 5^{x-1}$
13. $g(x)=g(x-1)+3$

$$
g(1)=1
$$

14. $h(x)=-5(x-1)+2$
15. List everything you know about Arithmetic Sequences.
16. List everything you know about Geometric Sequences.

## Part 4

(a) Determine whether the following are linear, exponential, or neither.
(b) Determine whether each relationship is continuous or discrete
(c) Determine the domain and range of each relationship
1.

2.

| Rounds | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Number of <br> players left | 64 | 32 | 16 | 8 | 4 |

3. $y=2(5)^{x}$
4. 



$$
y=4 x+3
$$

5. A water purification plant just installed a new pump that cleanses 4 gallons of water per minute. Suppose the plant already had 500 gallons of pure water when they replaced the pump and that the pump runs all day every day.
a) Create a table
b) Create a graph
c) Create an explicit function
d) Explain each piece of your function in part (c)
e) Is the relationship linear or exponential?
f) Is the relationship discrete or continuous?
g) Is the relationship a sequence? Why or why not? If so, what type?
6. When Facebook was founded it had three initial users. Now, since so many people use Facebook, they use computers to calculate the total users. At the end of each month, at midnight, the computers determine the number of users. Looking back on past data, they've noticed that the number of users increased by $75 \%$ each month.
a) Create a table
b) Create a graph
c) Create an explicit function
d) Explain each piece of your function in part (c)
e) Is the relationship linear or exponential?
f) Is the relationship discrete or continuous?
g) Is the relationship a sequence? Why or why not? If so, what type?

Match the equations and functions in the first column with the equivalent equation in the second column.
7. $y=\frac{3}{4} x+5 \quad$ a. $f(0)=2, f(n)=f(n-1) \cdot 5$
8. $2 x+5 y=10$
b. $y-7=3(x-2)$
9. $y=2 \cdot 5^{x}$
c. $f(0)=2, f(n)=f(n-1)-\frac{2}{5}$
10. $f(x)=3 \cdot 4^{x-1}$
d. $y=-2 x+6$
11. $6 x+3 y=18$
e. $y-8=\frac{3}{4}(x-4)$
12. $f(x)=3 x+1$
f. $f(1)=3, f(n)=f(n-1) \cdot 4$
13. What are the requirements for a relationship to be a sequence?
14. Determine whether the following relationships are sequences or not. If a sequence, state what type.
(a)

| $x$ | $F(x)$ |
| :---: | :---: |
| -2 | 3 |
| -1 | 5 |
| 0 | 7 |
| 1 | 9 |
| $\ldots$ | $\ldots$ |

(c)

| $x$ | $F(x)$ |
| :---: | :---: |
| 0 | 3 |
| 0.5 | 8 |
| 1 | 13 |
| 1.5 | 18 |
| $\cdots$ | $\ldots$ |


| $x$ | $F(x)$ |
| :---: | :---: |
| 0 | 3 |
| 1 | 12 |
| 2 | 21 |
| 3 | 30 |
| $\ldots$ | $\cdots$ |

Let $f(x)=4(6)^{x}$ and $g(x)=7 x-10$
15. Is $f(x)$ linear or exponential? Create a table for $f(x)$ below:

| x | $\mathrm{f}(\mathrm{x})$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

16. Is $g(x)$ linear or exponential? Create a table for $g(x)$ below:

| x | $\mathrm{g}(\mathrm{x})$ |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

17. Create a graph that fits the following descriptions to help you answer \#18 below.

| (A) Create a graph with both a <br> linear increasing function, and <br> an exponential increasing <br> function: | (B) Create a graph with both a <br> linear increasing function, and <br> an exponential decreasing <br> function: | (A) Create a graph with both a <br> linear decreasing function, and <br> an exponential decreasing <br> function: |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

18. Which relationship, out of linear or exponential, will have a greater output in the long run?
19. Determine whether the following relationships are linear, exponential, or neither.
(a)

| $x$ | $f(x)$ |
| :---: | :---: |
| -1 | 4 |
| 1 | 8 |
| 4 | 2 |
| 5 | 6 |
| 9 | 30 |

(b)

| $x$ | $f(x)$ |
| :---: | :---: |
| 0 | 3 |
| 1 | 6 |
| 3 | 24 |
| 7 | 384 |
| 9 | 1536 |

(c)

| $x$ | $f(x)$ |
| :---: | :---: |
| 0 | -6 |
| 1 | -12 |
| 2 | -24 |
| 4 | -48 |
| 6 | -96 |

20. Write the equation of $f(x)$ shown in the graph below.
a) Then find the average rate of change of $f(x)$ when $x$ is between 1 and 4 .
b) Draw in the secant line (the line that you are finding the slope of when $x$ is between 1 and 4).
21. Write the equation of $g(x)$ shown in the graph below.
a) Then find the average rate of change of $g(x)$ when x is between 0 and 2 .
b) Draw the secant line (the line that you are finding the slope of when $x$ is between 0 and $2)$.

22. Let $h(x)=3 \cdot 7^{x}$ and find the average rate of change of $h(x)$ when x is between and -1 and 5 .

## Part 5

List Key Features of the following functions:
1.

2.

3.

4.

5. What is the definition of a function?
6. Give an example of a function in the following representations:
(a) Table
(b) Ordered Pairs
(c) Map
(d) Graph
(e) Equation
7. Give an COUNTER EXAMPLE of a function in the following representations:
(a) Table
(b) Ordered Pairs
(c) Map
(d) Graph
(e) Equation

Given the descriptions below, sketch a possible graph of the function. There is more than one possible correct answer.
8.
a. Domain contains all Real numbers between -2 and 3.
b. Range contains all Real numbers between 3 and 7.
c. The function is increasing from -2 to 0 and decreasing after 0 .

d. The function is not continuous at every point.
9.
a. The function has a minimum at -5 .
b. The function has a maximum at 8 .
c. The function has two intervals on which it is decreasing and one interval on which it is increasing.
d. The Domain of the functions contains all Real numbers from 1 to 9 .

10.
e. This function is not continuous anywhere.
f. The function contains only seven elements in its domain.
g. The values of the domain are between -10 and 2.
h. The values of the range are between -1 and 1 .

11. List the key features of a linear function
12. List the key features of an exponential function
13. List the similarities and differences between the key features of linear and exponential functions.

Matching I - Match each image with the transformation that has taken place. (G.CO.4)
1.

2.

3.

4.

(A) Reflect across $y=-2 x+1$
(C) Rotate $90^{\circ}$ Clockwise around the point $(0,0)$
(B) $f(x, y) \rightarrow(x+5, y+6)$
(D) $f(\mathrm{x}, \mathrm{y}) \rightarrow(\mathrm{x}+1, \mathrm{y}+2)$
5. What transformations preserve distance and angles between the image and preimage?
6. What transformations DO NOT preserve distance and angles between the image and pre-image?
7. Based off questions $4-5$, which of the following transformations will preserve distances and angles between pre-image and image? (G.CO.2)
(A) $f(x, y) \rightarrow(4 x, y-4)$
(B) $f(x, y) \rightarrow(-3 x, y)$
(C) $f(x, y) \rightarrow(2-x, 2 y)$
(D) $f(\mathrm{x}, \mathrm{y}) \rightarrow(\mathrm{x}+2, \mathrm{y}-2)$
8. Which of the following transformations will not preserve distances and angles between pre-image and image? (G.CO.2)
(A) $f(x, y) \rightarrow(x+3, y-2)$
(B) $f(x, y) \rightarrow(-x, y)$
(C) $f(x, y) \rightarrow(3 x, 3 y)$
(D) $f(\mathrm{x}, \mathrm{y}) \rightarrow(\mathrm{x}, \mathrm{y}+6)$

Perform the requested transformation. If you transform point $A$, make sure you label the transformed point as $A^{\prime}$. (G.CO.5)
9.

$f(x, y) \rightarrow(x-4, y-3)$
11.


Rotate the line segment $\mathrm{AB} 90^{\circ}$ counterclockwise around the point $(1,1)$
10.

$f(x, y) \rightarrow(x-5, y+2)$
12.


Clearly draw the line of reflection on the graph above. Write the equation of the line.
13.


Clearly draw the line of reflection on the graph above. Write the equation of the line.
14.


Graph a perpendicular line to the one shown above. Write the equation of both lines.
15. What are the defining features of a mathematical reflection? Explain what is necessary for a transformation to be precisely a reflection and what you know about the pre-image and image.
16. What are the defining features of a mathematical rotation? Explain what is necessary for a transformation to be precisely a rotation and what you know about the pre-image and image.
17. What are the defining features of a mathematical translation? Explain what is necessary for a transformation to be precisely a translation and what you know about the pre-image and image.
18. How are transformations like functions? (G.CO.2)

## Part 7

For questions 1-2 use the figure below.


1. List all the angles of rotation up to $360^{\circ}$ that will carry the figure onto itself. (G.CO.3)
2. On the figure above, draw the lines of reflection (symmetry) that carry the figure onto itself. (G.CO.3)
3. Determine if the figures below are congruent. Justify your answer. (G.CO.6)

4. Determine if the figures below are congruent. Justify your answer. (G.CO.6)

5. Suppose the following figures are congruent. List all of the corresponding parts that are congruent. (G.CO.7)

6. Determine if the following figures are congruent. Justify your answer. (G.CO.7)

7. Why can we use the ASA, SAS, and SSS rules to determine if two triangles are congruent? (G.CO.8) Give an argument for each and include illustrations to justify your answer.
8. Why does SSA fail to prove that two triangles are congruent? Give an argument and illustration to justify your answer. (G.CO.8)

## Part 8

1. If $f(x)=g(x)+6$, and $g(x)=4 x-2$, then $f(x)=$
2. If $f(x)=g(x)-3$, and $g(x)=9 \cdot 2^{x}$, then $f(x)=$
3. In the equation $\boldsymbol{h}(\boldsymbol{x})=\mathbf{3} \cdot \mathbf{2}^{x}+\mathbf{4}$ the y -intercept is $\qquad$ (write your answer as an ordered pair).

Use vectors $\vec{a}:\langle-1,2\rangle, \vec{b}:\langle-2,3\rangle$, and $\vec{c}:\langle 3,1\rangle$ to answer the following questions:
4. Find the solution to $2 \vec{a}-\vec{c}$ using both a coordinate grid AND components.

5. Find $\|\vec{a}+\vec{b}\|$
6. Find the determinant of the following matrix $A=\left[\begin{array}{cc}2 & 7 \\ 4 & 14\end{array}\right]$
7. Does A (from \#6) have a multiplicative inverse?
8. Find the multiplicative inverse of the following matrix using a system of equations. Check your work - you have a good way to do this easily. SO DO IT! ©
$B=\left[\begin{array}{ll}3 & 4 \\ 2 & 1\end{array}\right]$
9. Find the multiplicative inverse of the following matrix using any method you choose:
$C=\left[\begin{array}{cc}-2 & 5 \\ 4 & 3\end{array}\right]$
10. Solve the following system using SUBSTITUTION, ELIMINATION, or GRAPHING.
$\left\{\begin{aligned} 4 x+8 y & =20 \\ -4 x+2 y & =-30\end{aligned}\right.$
11. Solve the following system using ROW REDUCTION of an AUGMENTED MATRIX. Show your work. You may use a calculator to CHECK your work, but not solve it for you.
$\{-6 x+5 y=1$
$\{6 x+4 y=-10$
12. Solve the following system using an INVERSE MATRIX. Show your work. You may use a calculator to CHECK your work, but you need to show me what you typed in.
$\left\{\begin{array}{l}8 x+y=-16 \\ -3 x+y=-5\end{array}\right.$

1. The table shows the largest vertical drops of nine roller coasters in the U.S. and the number of years after 1988 that they were opened. Plot the points on the coordinate plane below. (S.ID.6)

| Years since 1988 | 1 | 3 | 5 | 8 | 12 | 12 | 12 | 13 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Vertical Drop (ft) | 151 | 155 | 225 | 230 | 306 | 300 | 255 | 255 | 400 |


2. With your calculator find the linear regression line and sketch it on your graph. (S.ID.6)
a. $y=13.95 x+127.41$
b. $y=23.77 x+134.17$
c. $y=13.86 x+127.58$
d. $y=15 x+120$
3. What does the slope represent in terms of the context? (S.ID.7)
a. Feet dropped
b. Years since 1988
c. Years since 1988 per one foot dropped
d. Feet dropped per one year
4. What does the y-intercept represent? (S.ID.7)
a. The predicted largest vertical drop ever
b. The predicted average vertical drop in 1988
c. The predicted largest vertical drop in 1988
d. The predicted average vertical drop in the beginning
5. Using your equation, predict the largest vertical drop in 2018. Show your work. (S.ID.6)
a. 543.38
b. 577.41 feet
c. 545.91 feet
d. 847.27 feet
6. Determine the correlation coefficient for the data set above.
a. -0.89
b. 0.89
c. -0.98
d. 0.98
7. What type of correlation exists between the year and the vertical drop? (S.ID.8)
a. Weak, Negative Correlation
b. Strong, Negative Correlation
c. Weak, Positive Correlation
d. Strong, Positive Correlation
e. No Correlation

The table below shows the relationship between the number of hours studied and the percentage scored on a test.

| Hours Studied | 6 | 2.5 | .5 | 4 | 3.25 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Test Percentage | 98 | 89 | 75 | 90 | 84 | 91 |

8. Calculate the linear regression line. (S.ID.6)
a. $y=3.70 x+74.73$
b. $y=-1.42 x+81.74$
c. $y=3.69 x+74.74$
d. $y=1.42 x+81.74$
9. TRUE or FALSE: (Choose A for True, B for False) According to the residual plot, the linear regression line is a good fit for this data set. (S.ID.6)
10. The relationship between the year and the largest vertical drop is a...(S.ID.9)
a. Correlation Relationship
b. Causation Relationship
11. The relationship between time studied and test percentage is a... (S.ID.9)
a. Correlation Relationship
b. Causation Relationship
